Multimaterial Incompressible Flow Simulation Using the Moment-of-fluid Method

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We have incorporated the moment-of-fluid (MOF) interface reconstruction method into a finite element-based, variable-density, incompressible Navier-Stokes solver for the simulation of fluid flows involving large numbers of different materials. This method provides a significant improvement in accuracy while eliminating the nonphysical material order dependence.

For multimaterial flow simulations, accurate and robust management of material interfaces is essential. A proven computational technique, the volume-of-fluid (VOF) method [1], tracks the fractional volumes of materials in each computational cell and is widely used. When the material interfaces are required they are recreated from the volume fraction data in a process known as interface reconstruction.

In the interface reconstruction process, the material interfaces are reconstructed on a cell basis using arbitrarily oriented lines. These are referred to as piecewise linear interface reconstruction (PLIC) methods. The original PLIC method developed by Youngs [2] defined the outward interface normal as the negative gradient of the volume fraction function. It is first-order accurate, but gives reliable results and is widely used. These methods and their second-order accurate extensions such as LVIRA [3], create different interfaces depending on the order in which materials are processed and can only resolve features larger than two to three mesh cells.

In contrast, the second-order-accurate MOF method [4], developed at LANL by V. Dyadechko and M. Shashkov, has subcell accuracy and is material-order independent. MOF tracks the material volumes and cell-wise first moments and then recreates the interfaces by minimizing the discrepancy in the first moment between the actual and the reconstructed interface while exactly matching the volume. The superior accuracy and robustness of the MOF method is shown in advection and incompressible fluid flow simulations.

The advection of scalar quantities and materials is frequently encountered in complex multiphysics applications. To demonstrate the performance of MOF for advection problems, a simulation with significant deformation of the materials was performed. In this test case described in [5], a four-material circle is placed in a time-dependent vortex given by a solenoidal velocity field. At time t = 2, the circle will be at maximum deformation. From there, the flow reverses and at time t = 4, it should exactly match the initial condition. As shown in Fig. 1, the superior accuracy of MOF is apparent. In addition, the PLIC method results show strong dependence on material ordering.

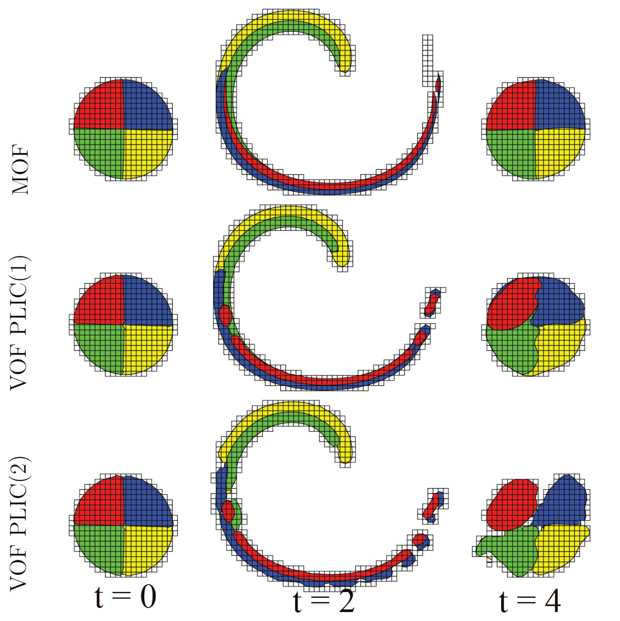

Fig. 1. Time-reversed vortex advection test. For VOF PLIC(1), the material order was (White, Red, Blue, Yellow, Green); for VOF PLIC(2) the order was (Red, Blue, Yellow, Green, White). At time t = 4, the material configuration should exactly match the initial condition at time t = 0.

For most problems of interest, the advection of material quantities impacts the evolution of the flow field. To explore these problems, we have incorporated MOF in a second-order accurate projection-based finite element method for numerical simulation of the variable density incompressible Navier-Stokes equations that describe the flow of immiscible, viscous fluids. In the simulation, the volume of each fluid in a cell is updated by geometrically solving an advection equation using interface reconstruction and a backward-Lagrangian technique.

As a demonstration, we consider the simple test case of two colliding bubbles where density differences drive the flow and the material interfaces and positions are strongly coupled to the flow field. The top and bottom bubbles have a density of 1.5 g/cm^3 and 0.5 g/cm^3 respectively and are placed in a fluid with a density of 1.0 g/cm^3. As shown in Fig. 2, MOF better preserves the thin material interfaces than the first-order PLIC method and the second-order LVIRA method.

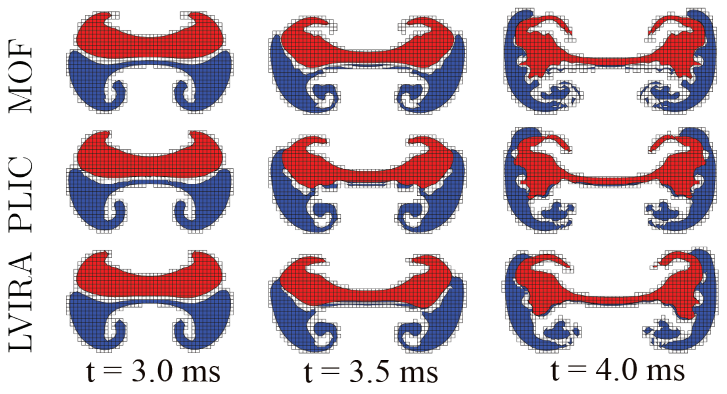

Fig. 2. Colliding bubbles simulations. The LVIRA and PLIC methods show stronger numerical surface tension and the breakup of the thin filaments. MOF preserves the thin white fluid layer separating the red and blue fluids.

By integrating MOF into a variable density incompressible flow solver, we have demonstrated the capability of MOF to provide more accurate and reliable material interface treatment in dynamic simulations. In such simulations, the MOF method clearly outperforms existing first- and second-order methods and represents a significant advance in material treatment.

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